# DISTURBANCE WAVES


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# 1. INTRODUCTION

- WORK THROUGH PAPER

  HALL-TAYLOR NS, HEWITT IJ, OCKENDON JR, WITELSKI, TP

  A NEW MODEL FOR DISTURBANCE WAVES

  INTER J MULTIPHASE FLOW <u>66</u> (2014), 38 – 45.

- GAS/LIQUID TWO-PHASE ANNULAR FLOW IN A TUBE

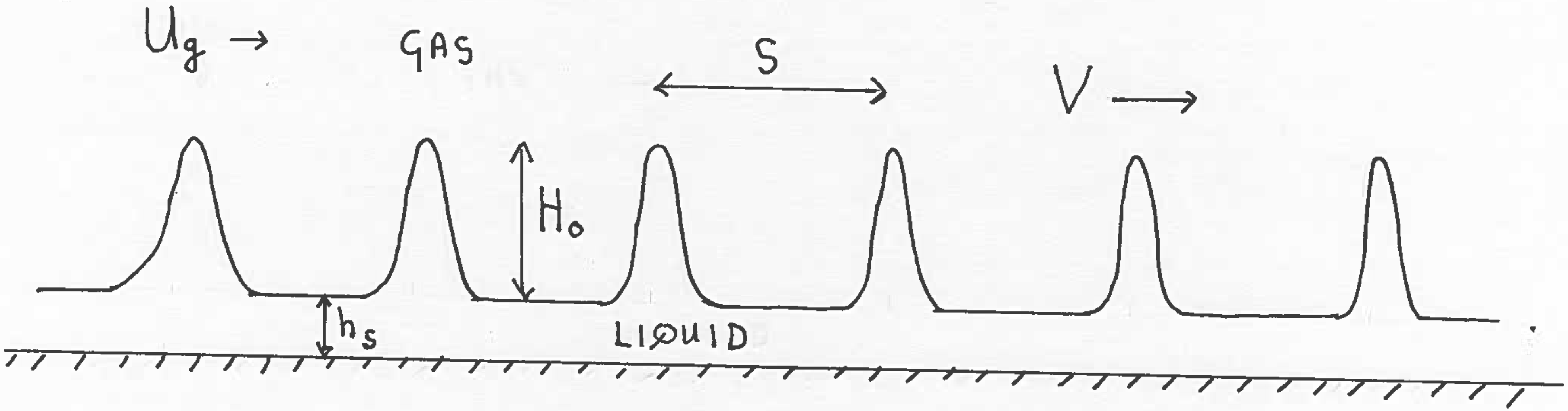
  BOILERS (WATER AND STEAM)

  EXTRACTION OF CRUDE OIL AND NATURAL GAS

## 2. PROPERTIES

GAS PHASE MOVES IN CORE OF TUBE

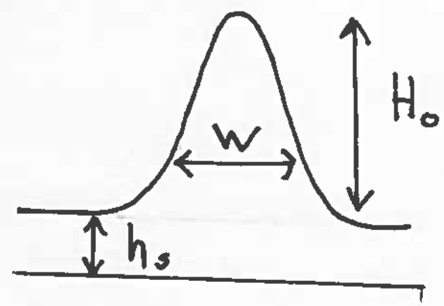
THIN LIQUID PHASE ON TUBE WALL




- UNIFORM WAVE SPEED $V$ $\qquad V \ll U_g \qquad \frac{V}{U_g} = O\left(\frac{1}{10}\right)$
- UNIFORM SEPARATION $S$

$S$ TWO OR THREE ORDERS OF MAGNITUDE GREATER THAN $H_o$

S INDEPENDENT OF $U_g$ AND LIQUID FLOW RATE




- LARGE AMPLITUDE

$$H_o \gg h_s$$

- SMALL ASPECT RATIO

$$\text{ASPECT RATIO} = \frac{\text{WIDTH}}{\text{HEIGHT}} = \frac{W}{H_o} \sim \frac{1}{100}$$

- INERTIA DOMINATES OVER GRAVITY

NEGLECT GRAVITY

- GAS REYNOLDS NUMBER $Re_g \gg 1$ (BASED ON TUBE RADIUS)

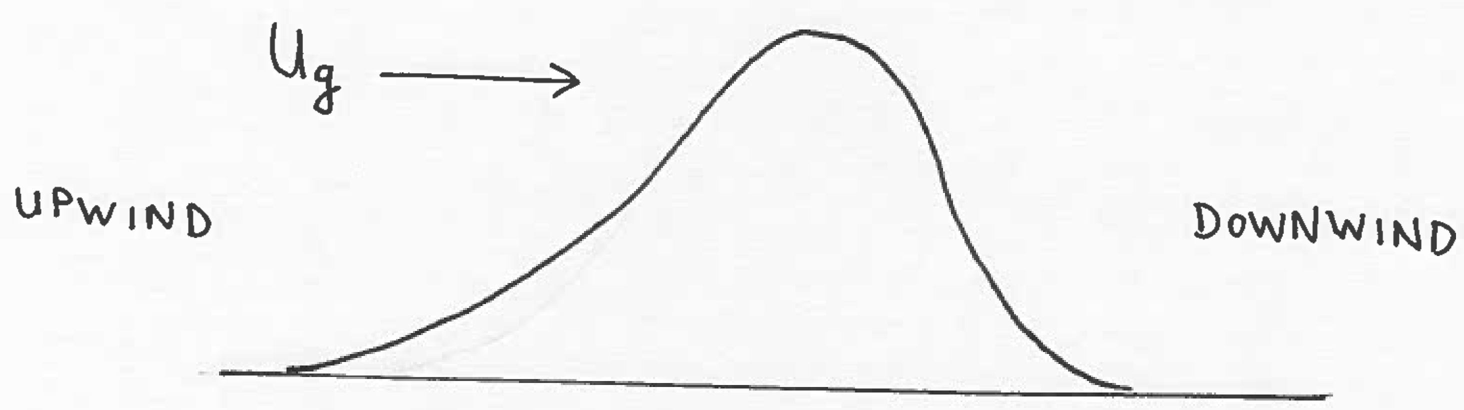
LIQUID REYNOLDS NUMBER $Re_\ell \gg 1$ (BASED ON FILMTHICKNESS)

NEGLECT VISCOSITY

- EFFECTS OF GAS COMPRESSIBILITY CAN BE NEGLECTED

INCOMPRESSIBLE FLUIDS

- SLOPE OF DOWNWIND FACE CHANGES MUCH MORE RAPIDLY THAN SLOPE OF UPWIND FACE



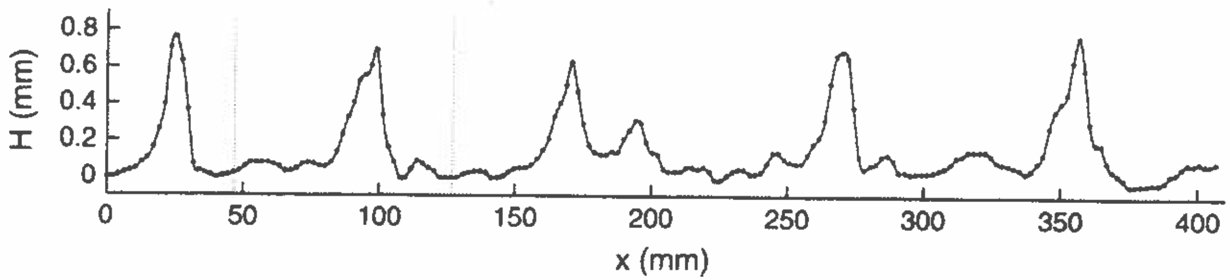



**Fig. 6.** Spatial wave profile showing film thickness above sub-layer. The waves are moving to the right. Vertical lines mark the boundaries between waves used for calculations in Section 4. Inferred from Wang et al. (2004).

# 3 MATHEMATICAL MODEL

- WORK IN FRAME MOVING WITH SPEED $V$ RELATIVE TO FIXED LABORATORY FRAME.

- PRANDTL - BATCHELOR THEOREM

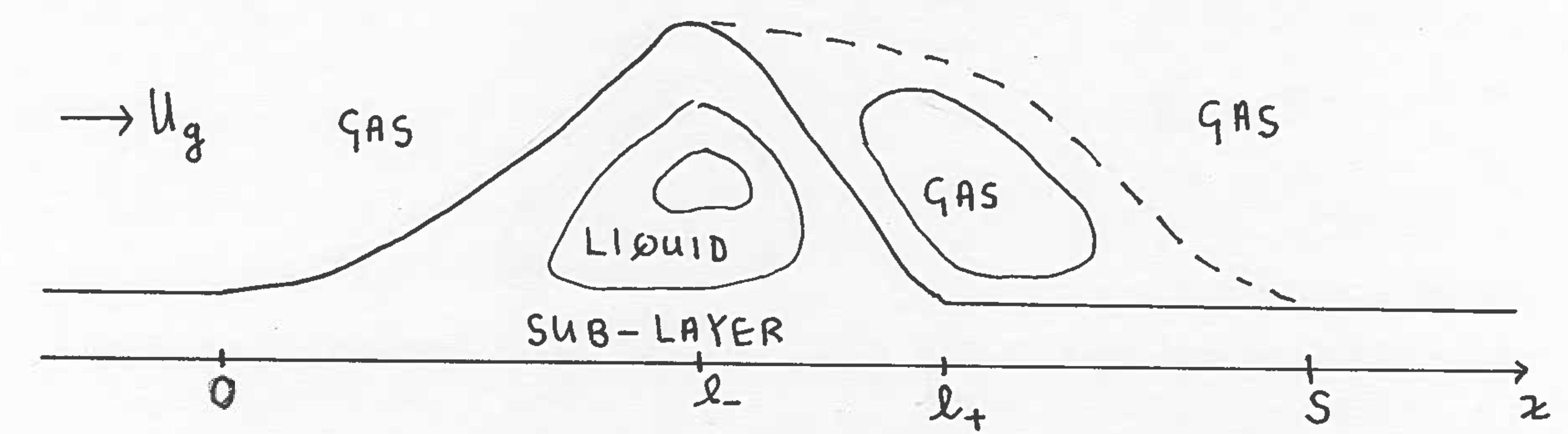
  IN STEADY TWO-DIMENSIONAL FLOW THE VORTICITY IS CONSTANT THROUGHOUT ANY REGION OF CLOSED STREAMLINES <u>IN THE LIMIT $\nu \to 0$</u> ($Re \to \infty$).

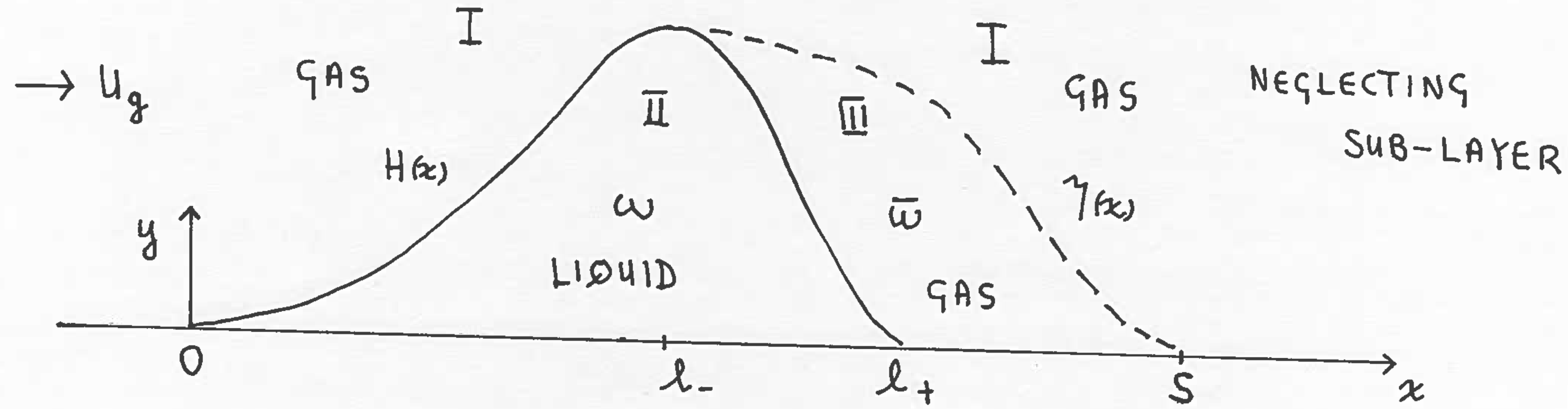
- BERNOULLI'S STREAMLINE THEOREM

  IF AN IDEAL FLUID ($\nu = 0$) IS IN STEADY FLOW AND THE BODY FORCE IS NEGLECTED THEN

$$p + \tfrac{1}{2}\rho V^2 = \text{CONSTANT ALONG A STREAMLINE.}$$

# SINGLE DISTURBANCE WAVE IN MOVING FRAME

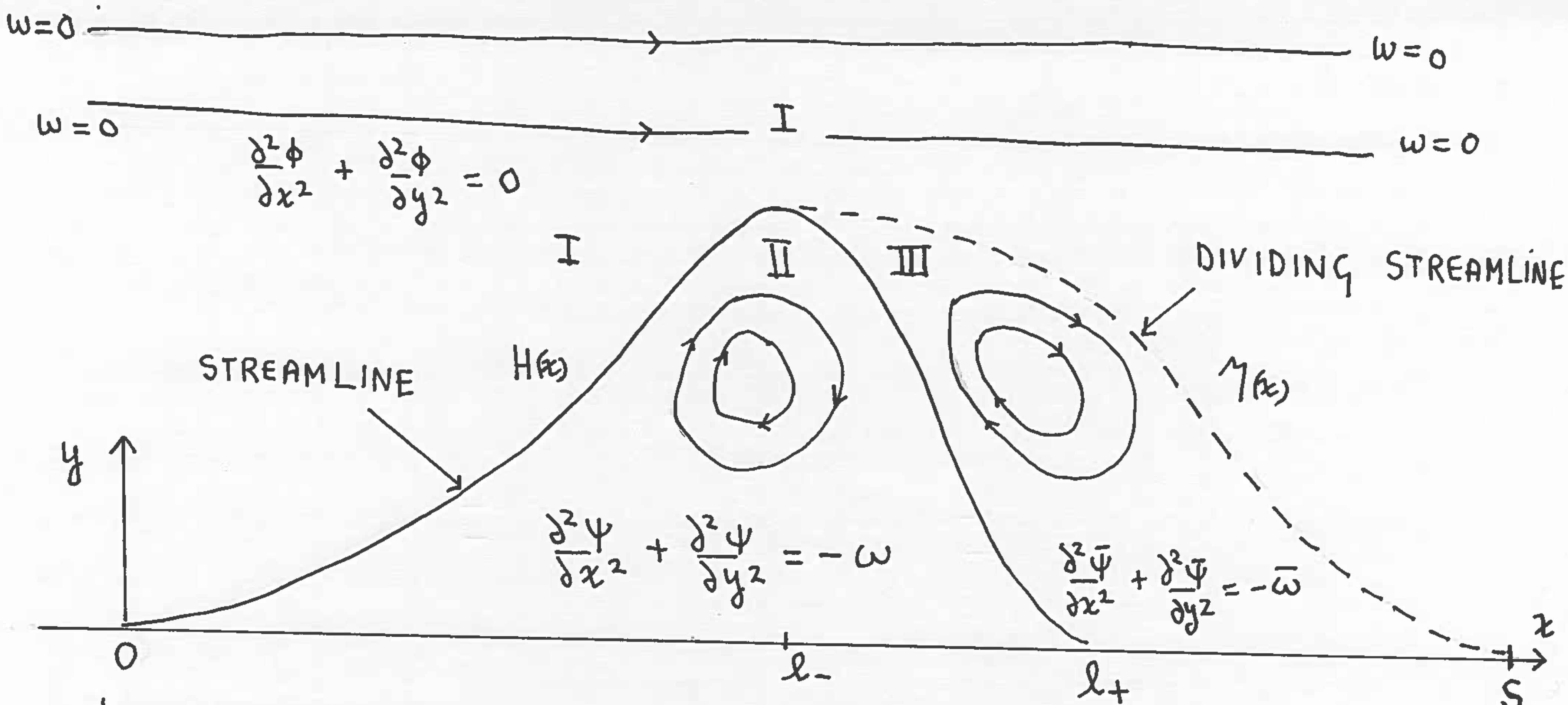



REGION I IRROTATIONAL GAS (POTENTIAL FLOW)

II CONSTANT VORTICITY LIQUID

III CONSTANT VORTICITY GAS




$\phi$ = VELOCITY POTENTIAL FOR GAS

$\psi$ = STREAM FUNCTION FOR INCOMPRESSIBLE LIQUID

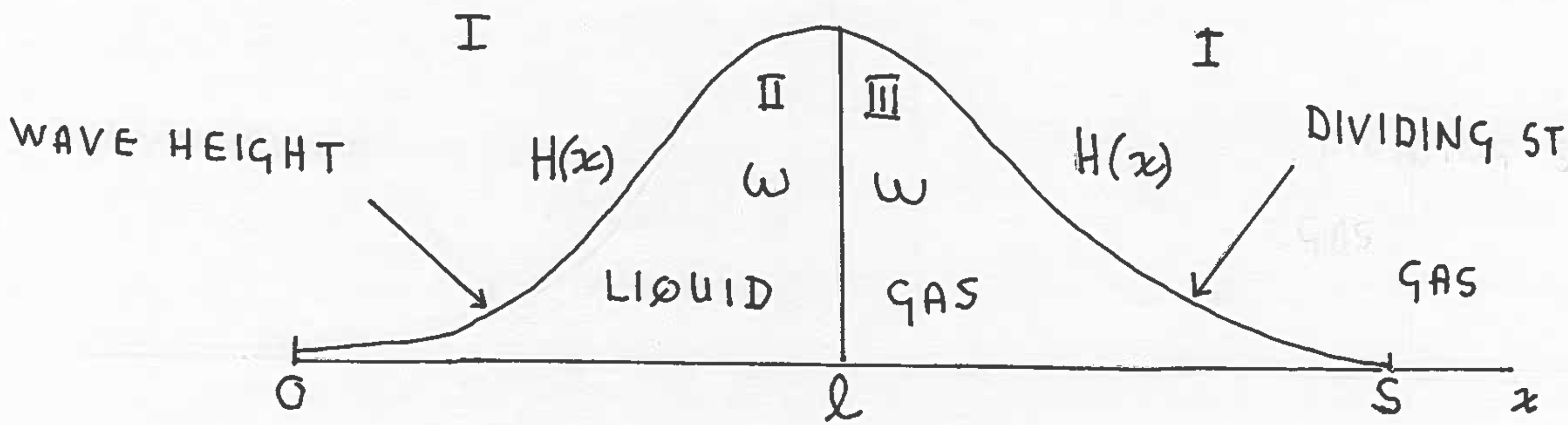
$\bar{\psi}$ = STREAM FUNCTION FOR INCOMPRESSIBLE GAS

BERNOULLI'S STREAMLINE THEOREM

BOUNDARY CONDITIONS

- MODEL WAVE IN LIMIT $\ell_- = \ell_+ = \ell$, $\bar{\omega} = \omega$

UNABLE TO FIND CONVERGENT NUMERICAL SOLUTION WHEN $\bar{\omega} \neq \omega$




INTERFACE $H(x)$ SATISFIES FREDHOLM INTEGRAL EQUATION OF SECOND KIND

$$\int_0^1 \frac{dH}{d\xi} \cot \pi(x-\xi)\, d\xi = -P + \frac{1}{8}\omega^2 H^2(x)$$

KERNEL SINGULAR AT $\xi = x$

NEGOTIATE SINGULARITY BY USING SYMMETRIC DISCRETIZATION $\xi_j = \frac{1}{2}(x_j + x_{j+1})$

BOUNDARY CONDITIONS $H(0) = 1$, $\frac{dH}{d\xi}(0) = 0$ DETERMINE $P$ AND $\omega$

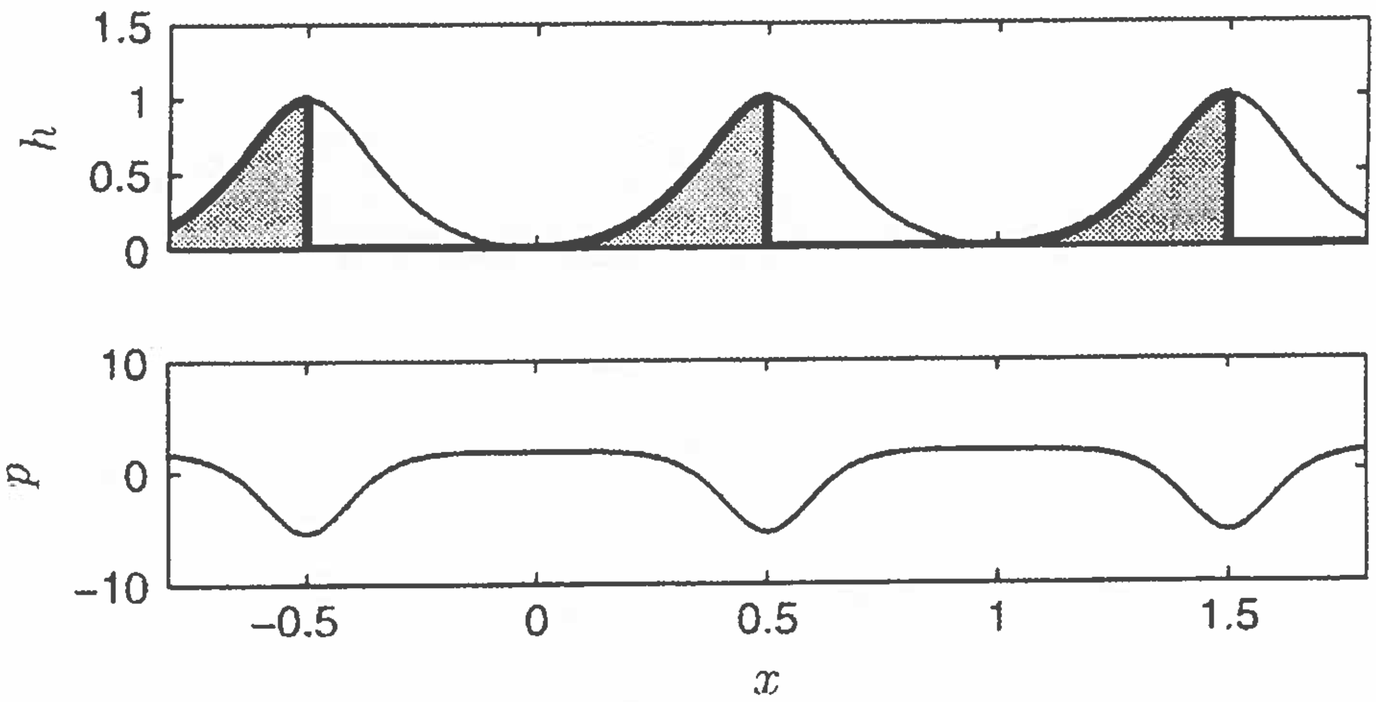



**Fig. 10.** Numerical solution of (48) for $H(x)$ with $H(\frac{1}{2}) = 1$, and with the interface between liquid and wake taken to lie where the pressure gradient changes sign at $\ell = \frac{1}{2}$. The lower panel shows the corresponding pressure from (43). The corresponding Bernoulli constant is $P_* \approx 1.81$, and the vorticity is $\omega_* \approx 7.62$.